# Increasing quantum computational efficiency through Quantum Error Correction codes benchmarking

## Introduction

The electron possess higher energy level, as long as it remains in the excited state. As the excited states are unstable, electron jumps back to its original level, and emits a quantum of light – photon, which energy is directly proportional to its frequency. and non-unitary processes such as measurements and environmental interactions. When such a situation was not predicted by a numerical models for quantum mechanics, such as unitary processes like quantum tunneling, governed by the Schrödinger equation, it can be generalized to be caused by noise and interference from an environment, and encoded as non-unitary process. Insufficient qubits isolation from an environment is a main drawback in NISQ solutions, and limits scaling capabilities – quantum computers utilized to simulate quantum systems, generally operate on up to 10 logical qubits, while for more complex, specific cases operate on up to 40 logical qubits.[[1]](#footnote-1)

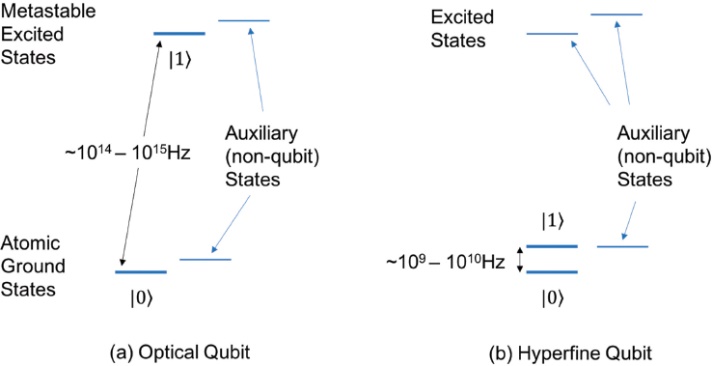


Figure 1. Schematic representation of two types of Optical Qubits and Hyperfine Qubits. Part (a) illustrates an optical qubit, which is formed by pairing an atomic ground state with a metastable excited state, with a frequency separation in the range of approximately 10^14 to 10^15 Hz. Part (b) shows a hyperfine qubit, composed of two atomic ground states with a frequency difference ranging from about 10^9 to 10^10 Hz.[[2]](#footnote-2)

While the number of mechanical solutions to mitigate this limitation is growing, and therefore – number of physical qubits can be potentially enlarged to hundreds and thousands of them[[3]](#footnote-3), they need faultless control that will allow to reliably operate them, and mitigating occurring errors. One of the classical examples of such mechanisms is Shor code, which encodes 1 logical qubit in 9 physical qubits, and can correct for arbitrary errors in single qubit. The work on that mechanism was published in 1995, and since then - a lot of physical limitation were overcome. The error threshold for Shor code is of 0.3, which is relatively high compared to other QECCs. In 1998 Alexei Kitaev and Sergey Bravyi introduced quantum error-correcting surface code, which after many rounds of improvements, have error threshold close to 0.018. Lower error threshold implies possibility of utilizing larger-scale quantum processing architectures, but still too small as for practical use. Surface codes with distances of 3 and 5 (with 9 - 25 qubits) have been practically implemented so far, while higher distance surface codes of up to d=25 are discussed theoretically in the literature.[[4]](#footnote-4)

Nowadays, scientists are working on another class of quantum-adapted, error correcting codes called LDPC - Low-density parity-check code, invented in 1965 gained attention at the beginning of 00s for their near-Shannon Capacity performance, making them highly relevant for NASA missions. These codes are characterized by their high coding rates and moderate code lengths, with the ability to correct errors efficiently, as indicated by their large minimum distances and low error floors.[[5]](#footnote-5) Their quantum variant beat the current golden standard, a surface code, by orders of magnitudes. The principles of LDPC codes can be adapted to the quantum domain to create Quantum LDPC (QLDPC) codes. These codes are designed to correct quantum errors by using a sparse parity-check matrix, similar to their classical counterparts. The sparsity of the matrix is crucial because it allows for efficient decoding algorithms, which is particularly important in quantum systems where the computational overhead needs to be minimized due to the delicate nature of quantum states.

Obraz zawierający tekst, Wykres, linia, diagram

Opis wygenerowany automatycznie

Figure 2. QLDPC decoder's runtime (labeled GD) is significantly higher than that of the heuristic decoder (labeled UFH), especially as the code size increases. This is due to the general decoder's reliance on Gaussian elimination, which becomes a computational bottleneck for larger code sizes, and becomes a challenge for physical scaling of the computational unit.[[6]](#footnote-6)

The challenge lies in adapting QLDPC codes to function effectively within the constraints of actual quantum devices. Furthermore, the effectiveness of QECC depends on the computational architecture of the quantum system, which varies according to the type of qubit used—be it superconducting qubits, trapped ion qubits, or photonic qubits. For instance, neutral atom qubits, which are well-suited for creating long-range connections, exhibit different physical properties compared to architectures that primarily involve local connections, leading to different behaviors and error-correction requirements.

Although significant improvements have been performed in the field of QECCs, a unified methodology for evaluating them for specific conditions (such as QPU architecture) on a consistent basis has remained elusive. This reason pushed further development of benchmarking suites in the industry, and became origin of this article.

## (Quantum) Error Correction Codes – Behind the Scenes

The purpose of applying correction codes is easily noticeable once we consider transmission of an information through a noise channel. If Alice sends a single bit, the probability of Bob receiving the wrong bit against physical probability of error p is linear, as presented in the figure 1.

Obraz zawierający tekst, zrzut ekranu, linia, Wykres

Opis wygenerowany automatycznie

Figure 3. Probability of error on transmitted bit of information.

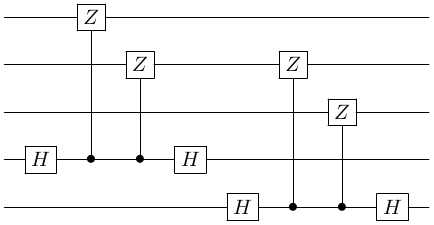
Another implication is that when the physical error rate p is low, the repetition code is beneficial as it can help correct errors effectively by repeating the information multiple times, allowing for error detection and correction. However, when the physical error rate p is large, using the repetition code can be less effective than the naive process. This is because in the presence of a high error rate, the repetition code's strategy of repeating information may lead to compounding errors, making it harder to distinguish the original message from the corrupted data. This implies the results presented in GHZ benchmarking, where due to the noise the overall efficiency of the dynamic version implementation of the algorithm decreased, in comparison to its static version.

The linear codes provides the ground for both classical and quantum error-correcting processes.[[7]](#footnote-7) As some concepts are common for both quantum and classical error-correction codes, such as distance, which in classical variant is the number of places where two bit strings have different symbols, allow to label correcting code by [n, k, d] for both realms of information, where d is the distance, k dimension of the message space, and n – dimension of the codespace. Hamming code, specific to classical counterpart, allows to detect defect bits of information by parity calculation on auxiliary bits. That mechanism can be reproduced in quantum mechanics realm, but requires specific notation, new symbolism, and extension.

Quantum operations, which are transformations applicable to the density matrices of quantum systems, encompass a broader range of processes than change of information state. While the Schrödinger equation governs the unitary evolution of quantum states in isolated systems - Hermitian operators, specific class of linear operators that correspond to observable physical quantities, quantum operations also account for non-unitary processes such as measurements and interactions with the environment. These operations are mathematically represented by completely positive, linear maps that act on density matrices - non-Hermitian processes that can change the trace of a density matrix, reflecting the probabilistic outcomes of measurements or lossy interactions with an environment.

These mechanism implies several quantum-specific behaviors, such as the no-cloning theorem, which prohibits the copying of quantum states, limits the types of operations that can be performed during the decoding process. Measuring quantum superposition on the other hand, destroys the superposition, preventing the simple measurement of received blocks to determine their state. Moreover, classical codes only need to address bit-flip errors, but quantum codes must handle a variety of unitary and non-unitary noisy interactions, including phase errors and continuous rotations, among others.

Fortunately, number of mitigation techniques were developed to create efficient quantum correcting codes. For example, non-destructive measurements of M, where M can be one-qubit or multi-qubit operator, allow for adapting repetition code for bit-flips. This code doesn’t mitigate errors caused by phase-flips however, and implies need of error correction codes redefinition specifically to quantum mechanics. Keeping the analogy to bit-flip error, it is entitled as X-axis error, due to analogy of applying X Pauli gate to a qubit being affected. Phase-flips, on the other hand, are encoded with Z-axis gates. While error correction codes for bit-flips may not apply for phase-flips errors, a more general approach has to be developed. One of the most famous addressing of this topic is CSS (Calderbank-Shor-Steane) construction, which relates quantum codes to classical linear codes over binary and quaternary fields.[[8]](#footnote-8) One of the examples for a well-designed code, which ensures that every possible error moves the state out of the code and into other subspace of the codespace, is Shor code.



Stabilizer codes are the simplest class of quantum error-correcting codes. These are defined as a class of quantum codes, which message space is the 2^k-dimensional Hilbert space, being a subspace of the 2^n-dimensional Hilbert space, labeled as [[n,k]].

## Existing frameworks

Although it is possible to implement correcting codes specifically each time when it is needed, as the systems are growing, it becomes impractical – same as in the nowadays, classical systems engineers do not build up cryptographical systems from scratch, but relies on already working systems and out-of-the-box that perform some of the automation for them.

One of the libraries that offers creation of such codes is qiskit-qec[[9]](#footnote-9). Qiskit Framework for Quantum Error Correction offers a stack for creating own QECC, with creating sympletic matrixes, providing stabilizer generators as strings, or utilizing already implemented codes with *CodeLibrarian* Class.

“MQT QECC: A tool for Quantum Error Correcting Codes written in C++”[[10]](#footnote-10), is an attempt to create framework-agnostic tooling for QECCs generation.

## Benchmarking methodologies

How scientists could they rely on their results, and what is their compass to indicate whether they are moving toward a good direction? The key here is benchmarking. To validate whether a particular correction code is working properly, it’s needed to properly select a stack of applications. The decisions criteria may be different, and the benchmarking suite may vary as well, as different applications can have different sensitivity for parameters changes. For example, even though Shor’s algorithm is one of the most popularized due to it’s evidence from upcoming quantum supremacy, or HHL algorithm – best known for its appliance within space radars sector, they may not be suitable for benchmarking. Moreover, Shor’s algorithm requires around 4098 logical qubits, what make it impossible to test it against real RSA-2048 cypher.

The purpose of Quantum Error Correction techniques is to provide operational reliability, aiming at Tera QuOps efficiency in the future. The architecture behind QEC stands on stabilizer codes, where stabilizer group S is an abelian subgroups of the Pauli group and therefore can be expressed with Pauli Operators. The minimal set of Pauli operators is a generator, and can create the full stabilizer group. Some of the most recognizable stabilizer codes are toric code and surface code.

Toric code is a topological QEC code, defined on a 2D lattice, with periodic boundary conditions, where stabilizer operators are defined on X-type vertices, and Z-type faces of the lattice, working on qubits located on the edges. Logical operations on the encoded qubits are performed by string-like operators that wrap around the torus.

Surface code is a variant of the toric code, with open boundary conditions – instead of periodic. As the surfaces codes can be implemented with simpler geometries than the toric code, they are easier for practical applications.

Triangular Color Codes is a variant of the color code quantum error-correcting code defined on a hexagonal lattice with three boundaries, which offers advantages in terms of fault-tolerant quantum computation compared to other topological codes.

## Benchmarking

When creating a benchmarking suite to evaluate the noise suppression efficiency of toric and surface codes, following parameters can be used as depended variables:

|  |  |
| --- | --- |
| Lattice size (based on code distance *D*) | Correlate with error resilience[[11]](#footnote-11) |
| Boundary condition | Open/Periodic |
|  |  |

Potential output

|  |  |
| --- | --- |
| Worst-Case Error Rate | Whether the codes meet the requirements for fault tolerance |
| Noise models | Evaluate codes’ performance under Pauli dephasing and depolarizing |
| Randomized compiling |  |

To make the comparison between surface code and toric code efficient, additional normalization has to be performed, as they have different rates. A source of inspiration is presented in “Practical fault-tolerant quantum computing”[[12]](#footnote-12) and “Threshold error rates for the toric and surface codes”[[13]](#footnote-13)

Tailoring Noise

Scalability: Studying how the performance of mid-circuit measurements scales as the circuit depth and size increase.

Realization: Evaluating the practical feasibility of implementing mid-circuit measurements using the QECC.

Complexity: Assessing the overall complexity of the QECC when integrated with mid-circuit measurement capabilities.

## Summary

Although attempts to use Quantum Error Correction codes as the application for benchmarking suite was already addressed[[14]](#footnote-14), combining them as a subcategory of application in overall benchmark for mid-circuit measurement is yet to be done.

One of the authors prepared benchmarking suite dedicates for surface code, together with a paper that presents benchmarking from various perspectives. The benchmark allows to analyze the qubit overhead required to perform mid-circuit measurements without compromising the overall error correction capabilities of the QECC, measure the maximum error rate that the QECC can tolerate while still effectively correcting errors introduced by mid-circuit measurements; assessing the QECC's ability to detect and correct the specific types of errors that may arise from mid-circuit measurements; investigating the efficiency and compatibility of decoding algorithms that can handle the additional syndromes introduced by mid-circuit measurements, and analyzing the impact of mid-circuit measurements on the availability and complexity of transversal gates.

In terms of

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